Integrable bootstrap for AdS_3/CFT_2 correlation functions

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based on 2102.08365 with Burkhard Eden and Alessandro Sfondrini

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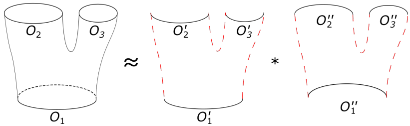
Hexagon tessellations in AdS_5

- **Spectrum** is fairly well-understood
- Three-point functions by **hexagon operators**

  [Basso, Komatsu, Vieira '15]



- In principle:
  - → **higher-point functions** [Eden, Sfondrini '17] [Fleury, Komatsu '17]
  - → **non-planar correlators** [Eden, Jiang, DℓP, Sfondrini '17] [Bargheer, Caetano, Fleury, Komatsu, Vieira '17] [Bargheer, Coronado, Vieira '19] ...
  - → **wrapping corrections** [see Basso's talk]

Can we study another (simpler?) setting?

AdS_3 string theory

String theory in AdS_3 has a richer structure than in AdS_5.
$\rightarrow$ There are **two** main **parameters**: $k \sim$ NSNS flux, $h \sim$ RR flux

Spectrum is classically integrable and believed to be at quantum level

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Special cases:

- pure RR: $k = 0$, $h \neq 0$ $\rightarrow$ similar to AdS_5
- pure NSNS: $k \neq 0$, $h = 0$ $\rightarrow$ spectrum is given by WZW model

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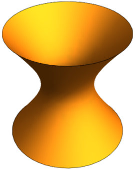
- For $k = 1,\ h = 0$ the WZW becomes a free theory

[Eberhardt, Gaberdiel, Gopakumar '18] ...

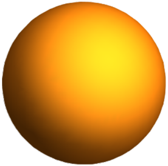
- For $h = 0$ the TBA is known for all k and wrapping is simple

Outline

Supersymmetry algebra



AdS_3



S^3

$$\mathfrak{so}(2,2) = \mathfrak{su}(1,1) \oplus \mathfrak{su}(1,1)$$

$$\mathfrak{so}(4) = \mathfrak{su}(2) \oplus \mathfrak{su}(2)$$

The bosonic subalgebra is then $[\mathfrak{su}(1,1) \oplus \mathfrak{su}(1,1)] \oplus [\mathfrak{su}(2) \oplus \mathfrak{su}(2)]$.

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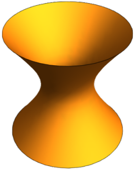

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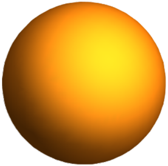

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add supersymmetry $\quad \to \quad \mathfrak{psu}(1,1|2)_L \oplus \mathfrak{psu}(1,1|2)_R$

Two copies of $\mathfrak{su}(1,1|2)$

	Left copy $\mathfrak{su}(1,1|2)_{\mathsf{L}}$	Right copy $\mathfrak{su}(1,1|2)_{\mathsf{R}}$
Contribution to the light-cone Hamiltonian	$\mathbf{H} \equiv -\mathbf{L}_0 - \mathbf{J}_3\,.$	$\widetilde{\mathbf{H}} \equiv -\widetilde{\mathbf{L}}_0 - \widetilde{\mathbf{J}}_3\,.$
BPS bound	$\mathbf{H} \geq 0$	$\widetilde{\mathbf{H}} \geq 0$

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Half of supercharges commute with $\mathbf{H}$ or $\widetilde{\mathbf{H}}$	$\mathbf{Q}^1,\ \mathbf{Q}^2,\ \mathbf{S}_1,\ \mathbf{S}_2,$	$\widetilde{\mathbf{Q}}_1,\ \widetilde{\mathbf{Q}}_2,\ \widetilde{\mathbf{S}}^1,\ \widetilde{\mathbf{S}}^2$

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Algebra of commuting charges is

$$\left\{\mathbf{Q}^A,\, \mathbf{S}_B\right\} = \mathbf{H}\,\delta^A{}_B \quad \text{and} \quad \left\{\widetilde{\mathbf{Q}}_A,\, \widetilde{\mathbf{S}}^B\right\} = \widetilde{\mathbf{H}}\,\delta_A{}^B\,,$$

with central extension

$$\left\{\mathbf{Q}^A,\, \widetilde{\mathbf{Q}}_B\right\} = \mathbf{C}\,\delta^A{}_B \quad \text{and} \quad \left\{\mathbf{S}_A\,,\widetilde{\mathbf{S}}^B\right\} = \mathbf{C}^\dagger\,\delta_A{}^B\,.$$

Factorisation of the centrally extended algebra

$\mathfrak{psu}(1|1)^{\oplus 4}$ centrally extended $\longleftrightarrow$ similar to $\mathfrak{su}(2|2)^{\oplus 2}$ in AdS_5/CFT_4

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Introduce the $\mathfrak{psu}(1|1)^{\oplus 2}$ centrally extended algebra, given by

$$\left\{\mathbf{q},\, \mathbf{s}\right\} = \mathbf{H}, \quad \left\{\widetilde{\mathbf{q}}, \widetilde{\mathbf{s}}\right\} = \widetilde{\mathbf{H}}\,, \qquad \left\{\mathbf{q},\, \widetilde{\mathbf{q}}\right\} = \mathbf{C}, \quad \left\{\mathbf{s}, \widetilde{\mathbf{s}}\right\} = \mathbf{C}^{\dagger}\,.$$

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Larger algebra and relevant representations can be obtained by

$$\mathbf{Q}^{1}\equiv\mathbf{q}\otimes\mathbf{1}\,,\quad \mathbf{Q}^{2}\equiv\mathbf{1}\otimes\mathbf{q},\qquad \mathbf{S}_{1}\equiv\mathbf{s}\otimes\mathbf{1}\,,\quad \mathbf{S}_{2}\equiv\mathbf{1}\otimes\mathbf{s}\,,$$

and

$$\widetilde{\mathbf{Q}}_{1}\equiv\widetilde{\mathbf{q}}\otimes\mathbf{1}\,,\quad \widetilde{\mathbf{Q}}_{2}\equiv\mathbf{1}\otimes\widetilde{\mathbf{q}},\qquad \widetilde{\mathbf{S}}^{1}\equiv\widetilde{\mathbf{s}}\otimes\mathbf{1}\,,\quad \widetilde{\mathbf{S}}^{2}\equiv\mathbf{1}\otimes\widetilde{\mathbf{s}}\,.$$

Massless, left and right representations

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[Hoare, Stepanchuk, Tseytlin '13] [Lloyd, Ohlsson Sax, Sfondrini, Stefański '14]

k and h are properties of the string background

m plays the role of a mass in the dispersion relation

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k and h are properties of the string background

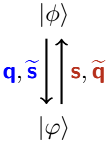
m plays the role of a mass in the dispersion relation

- **Massles**: $m = 0$
- **Left**: $m = +1, +2, \ldots$
- **Right**: $m = -1, -2, \ldots$

Short representations of $\mathfrak{psu}(1|1)^{\oplus 2}_{c.e.}$

Two dimensional short representation (one Boson and one Fermion)

$$|\phi\rangle = \text{highest-weight state}\,, \qquad |\varphi\rangle = \text{lowest-weight state}\,,$$



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$$\begin{array}{c} |\phi\rangle \\ \mathbf{q}, \widetilde{\mathbf{s}} \downarrow \uparrow \mathbf{s}, \widetilde{\mathbf{q}} \\ |\varphi\rangle \end{array}$$

By setting $|\phi\rangle$ to be a Boson or a Fermion, we get

$$\phi \to \phi^{\mathrm{B}} \equiv \text{Boson}\,, \qquad \varphi \to \varphi^{\mathrm{F}} \equiv \text{Fermion}\,,$$

or viceversa

$$\phi \to \phi^{\mathrm{F}} \equiv \text{Fermion}\,, \qquad \varphi \to \varphi^{\mathrm{B}} \equiv \text{Boson}.$$

Particle content of the theory

Left representation

$$|Y\rangle = |\phi^{\mathrm{B}}_{\mathrm{L}} \otimes \acute{\phi}^{\mathrm{B}}_{\mathrm{L}}\rangle \,,$$

$$|\Psi^1\rangle = |\varphi^{\mathrm{F}}_{\mathrm{L}} \otimes \acute{\phi}^{\mathrm{B}}_{\mathrm{L}}\rangle \,, \qquad |\Psi^2\rangle = |\phi^{\mathrm{B}}_{\mathrm{L}} \otimes \acute{\varphi}^{\mathrm{F}}_{\mathrm{L}}\rangle \,,$$

$$|Z\rangle = |\varphi^{\mathrm{F}}_{\mathrm{L}} \otimes \acute{\varphi}^{\mathrm{F}}_{\mathrm{L}}\rangle \,.$$

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$$|Y\rangle = |\phi_L^B \otimes \acute{\phi}_L^B\rangle \,,$$
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$$|Z\rangle = |\varphi_L^F \otimes \acute{\varphi}_L^F\rangle \,.$$

Right representation

$$|\tilde{Z}\rangle = |\phi_R^F \otimes \acute{\phi}_R^F\rangle \,,$$
$$|\tilde{\Psi}^1\rangle = |\varphi_R^B \otimes \acute{\phi}_R^F\rangle \,, \qquad |\tilde{\Psi}^2\rangle = -|\phi_R^F \otimes \acute{\varphi}_R^B\rangle \,,$$
$$|\tilde{Y}\rangle = |\varphi_R^B \otimes \acute{\varphi}_R^B\rangle \,.$$

Particle content of the theory

Left representation

$$|Y\rangle = |\phi^{\mathrm{B}}_{\mathrm{L}} \otimes \acute{\phi}^{\mathrm{B}}_{\mathrm{L}}\rangle \,,$$
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$$|Z\rangle = |\varphi^{\mathrm{F}}_{\mathrm{L}} \otimes \acute{\varphi}^{\mathrm{F}}_{\mathrm{L}}\rangle \,.$$

Right representation

$$|\tilde{Z}\rangle = |\phi^{\mathrm{F}}_{\mathrm{R}} \otimes \acute{\phi}^{\mathrm{F}}_{\mathrm{R}}\rangle \,,$$
$$|\tilde{\Psi}^1\rangle = |\varphi^{\mathrm{B}}_{\mathrm{R}} \otimes \acute{\phi}^{\mathrm{F}}_{\mathrm{R}}\rangle \,, \qquad |\tilde{\Psi}^2\rangle = -|\phi^{\mathrm{F}}_{\mathrm{R}} \otimes \acute{\varphi}^{\mathrm{B}}_{\mathrm{R}}\rangle \,,$$
$$|\tilde{Y}\rangle = |\varphi^{\mathrm{B}}_{\mathrm{R}} \otimes \acute{\varphi}^{\mathrm{B}}_{\mathrm{R}}\rangle \,.$$

Massless representations

$$|\chi^{\dot{1}}\rangle = |\phi^{\mathrm{B}}_{0} \otimes \acute{\phi}^{\mathrm{F}}_{0}\rangle \,,$$
$$|T^{\dot{1}1}\rangle = |\varphi^{\mathrm{F}}_{0} \otimes \acute{\phi}^{\mathrm{F}}_{0}\rangle \,, \quad |T^{\dot{1}2}\rangle = |\phi^{\mathrm{B}}_{0} \otimes \acute{\varphi}^{\mathrm{B}}_{0}\rangle \,,$$
$$|\tilde{\chi}^{\dot{1}}\rangle = |\varphi^{\mathrm{F}}_{0} \otimes \acute{\varphi}^{\mathrm{B}}_{0}\rangle \,.$$

$$|\chi^{\dot{2}}\rangle = i|\phi^{\mathrm{F}}_{0} \otimes \acute{\phi}^{\mathrm{B}}_{0}\rangle \,,$$
$$|T^{\dot{2}1}\rangle = i|\varphi^{\mathrm{B}}_{0} \otimes \acute{\phi}^{\mathrm{B}}_{0}\rangle \,, \quad |T^{\dot{2}2}\rangle = -i|\phi^{\mathrm{F}}_{0} \otimes \acute{\varphi}^{\mathrm{F}}_{0}\rangle \,,$$
$$|\tilde{\chi}^{\dot{2}}\rangle = -i|\varphi^{\mathrm{B}}_{0} \otimes \acute{\varphi}^{\mathrm{F}}_{0}\rangle \,.$$

S-Matrix and dressing factors

The full S matrix can be obtained from a tensor product

$$\mathbf{S} = S \otimes \acute{S}.$$

[Borsato, Ohlsson Sax, Sfondrini, Stefański, Torrielli '13]

It can be determined from symmetries (up to overall prefactors Σ) consists of 16 blocks. [Borsato, Ohlsson Sax, Sfondrini '12]

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Left-right symmetry allows to introduce the following notation

- **Massive-massive**: $S^{\bullet\bullet}$ with $\Sigma^{\bullet\bullet}$ for $_{\text{LL}}$, $_{\text{RR}}$
  $\tilde{S}^{\bullet\bullet}$ with $\tilde{\Sigma}^{\bullet\bullet}$ for $_{\text{LR}}$, $_{\text{RL}}$
- **Massless-massless**: $S^{\circ\circ}$ with $\Sigma^{\circ\circ}$
- **Mixed-mass**: $S^{\bullet\circ}$ with $\Sigma^{\bullet\circ}$ for $_{\text{L}\circ}$, $_{\text{R}\circ}$
  $S^{\circ\bullet}$ with $\Sigma^{\circ\bullet}$ for $_{\circ\text{L}}$, $_{\circ\text{R}}$

There is a proposal for the dressing factor for pure RR and pure NSNS

[Borsato, Ohlsson Sax, Sfondrini, Stefański, Torrielli '13, '16]

Plan

Symmetries of the three-point function

Take 1/2-BPS operator $O(0)$ at $z = 0$

$\rightarrow$ want to construct *three* translated operators $O(z)$
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$$\mathbf{T}_{\kappa} = i\mathbf{L}_{-} + i\widetilde{\mathbf{L}}_{-} + \kappa\,\mathbf{J}_{-} + \kappa\,\widetilde{\mathbf{J}}_{-}\,.$$

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$$\mathbf{T}_{\kappa} = i\mathbf{L}_{-} + i\widetilde{\mathbf{L}}_{-} + \kappa\,\mathbf{J}_{-} + \kappa\,\widetilde{\mathbf{J}}_{-}\,.$$

Use $\mathbf{T}_{\kappa}$ to construct one parameter family of operators starting from $O(0)$

$$O_{t,\kappa} = e^{t\,\mathbf{T}_{\kappa}}\,O(0)\,e^{-t\,\mathbf{T}_{\kappa}}\,.$$

The hexagon subalgebra

Supercharges that commute with the supertranslation generator $\mathbf{T}_\kappa$

$$\mathcal{Q}_A = \mathbf{S}_A - \frac{i}{\kappa}\,\epsilon_{AB}\,\mathbf{Q}^B, \qquad \widetilde{\mathcal{Q}}_A = \widetilde{\mathbf{Q}}_A - i\kappa\,\epsilon_{AB}\,\widetilde{\mathbf{S}}^B\,,$$

with the anticommutation relations

$$\{\mathcal{Q}_A, \mathcal{Q}_A\} = 0, \qquad \{\mathcal{Q}_1, \mathcal{Q}_2\} = -\frac{i}{\kappa}\Big(\{\mathbf{S}_1, \epsilon_{21}\mathbf{Q}^1\} + \{\epsilon_{12}\mathbf{Q}^2, \mathbf{S}_2\}\Big) = 0\,.$$

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Moreover, we have

$$\{\mathcal{Q}_A, \widetilde{\mathcal{Q}}_B\} = -i\kappa\{\mathbf{S}_A, \epsilon_{BC}\widetilde{\mathbf{S}}^C\} - \frac{i}{\kappa}\{\epsilon_{AC}\mathbf{Q}^C, \widetilde{\mathbf{Q}}_B\} = -\frac{i}{\kappa}\epsilon_{AB}\Big(\mathbf{C} - \kappa^2\mathbf{C}^\dagger\Big),$$

$\mathbf{C}$ and $\mathbf{C}^\dagger$ are the central extensions of $\mathfrak{psu}(1|1)^{\oplus 4}$ and are not in $\mathfrak{psu}(1,1|2)^{\oplus 2}$.

Introducing the **central charge**

$$\mathcal{C} \equiv -\frac{i}{\kappa}\Big(\mathbf{C} - \kappa^2\mathbf{C}^\dagger\Big)\,.$$

Bootstrapping the hexagon form factor from symmetry

Bootstrap principle

Indicating the form factor of $\mathbf{h}$ with *any* state Ψ as $\langle \mathbf{h}|\Psi\rangle$, it follows that

$$\langle \mathbf{h}|\mathcal{Q}_A|\Psi\rangle = 0\,, \qquad \langle \mathbf{h}|\widetilde{\mathcal{Q}}_A|\Psi\rangle = 0\,, \qquad \langle \mathbf{h}|\mathcal{C}|\Psi\rangle = 0.$$

The vanishing of the central charge $\mathcal{C} \equiv -\frac{i}{\kappa}\left(\mathbf{C} - \kappa^2 \mathbf{C}^\dagger\right)$ implies $\kappa^2 = 1$.

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One-particle form factors are given by

$$\langle \mathbf{h}|Y_p\rangle = i\,\langle \mathbf{h}|Z_p\rangle, \qquad \langle \mathbf{h}|\tilde{Z}_p\rangle = -i\,\langle \mathbf{h}|\tilde{Y}_p\rangle, \qquad \langle \mathbf{h}|\chi_p^{\dot{A}}\rangle = i\,\langle \mathbf{h}|\tilde{\chi}_p^{\dot{A}}\rangle,$$

while the remaining form factors vanish,

$$\langle \mathbf{h}|\Psi_p^A\rangle = 0\,, \qquad \langle \mathbf{h}|\widetilde{\Psi}_p^A\rangle = 0\,, \qquad \langle \mathbf{h}|T_p^{A\dot{A}}\rangle = 0\,.$$

Two-particle states

Two-particle form factors

Fixed by the bootstrap principle to S matrix elements (up to scalar prefactors) found by [Borsato, Ohlsson Sax, Sfondrini '13]

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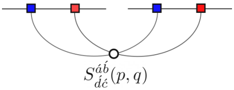
the **two-particle hexagon form factor** can be written as

$$\begin{aligned}\langle \mathbf{h}|\Phi_p^{a\acute{a}}\Phi_q^{b\acute{b}}\rangle &= \mathbf{K}_p\mathbf{K}_q(-1)^{(F_a+F_{\acute{a}})F_b}\left[|\xi_q^b\xi_p^a\rangle \otimes \mathbf{S}|\acute{\xi}_p^{\acute{a}}\acute{\xi}_q^{\acute{b}}\rangle\right] \\ &= (-1)^{(F_a+F_{\acute{a}})F_b}\, S^{\acute{a}\acute{b}}_{\acute{d}\acute{c}}(p,q)\mathbf{K}_p\mathbf{K}_q\left[|\xi_q^b\xi_p^a\rangle \otimes |\acute{\xi}_q^{\acute{d}}\acute{\xi}_p^{\acute{c}}\rangle\right].\end{aligned}$$



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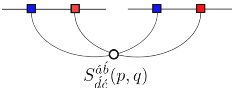


$\rightarrow$ fermionic statistics of massless one-particle hexagon form factors
$\Rightarrow$ Introduce the **contraction operator**

$$\mathbf{K}_p \equiv \sum_{a,\acute{a}} h^{a\acute{a}} \frac{\partial}{\partial \acute{\xi}^{\acute{a}}(p)} \frac{\partial}{\partial \xi^a(p)} \,.$$

Many-particle states

Proposal for the many-particle hexagon form factor

$$\left\langle \mathbf{h} \middle| \Phi^{a_1 \acute{a}_1}_{p_1} \Phi^{a_2 \acute{a}_2}_{p_2} \dots \Phi^{a_N \acute{a}_N}_{p_N} \right\rangle \equiv$$
$$\equiv (-1)^{F_{12\cdots N}} \mathbf{K}_{12\cdots N} \left[\left| \xi^{a_N}_{p_N} \dots \xi^{a_2}_{p_2} \xi^{a_1}_{p_1} \right\rangle \otimes \mathbf{S}_{12\cdots N} \left| \acute{\xi}^{\acute{a}_1}_{p_1} \acute{\xi}^{\acute{a}_1}_{p_1} \dots \acute{\xi}^{\acute{a}_N}_{p_N} \right\rangle \right].$$

where

$$F_{12\cdots N} \equiv \sum_{1 \le i < j \le N} (F_{a_i} + F_{\acute{a}_i}) F_{a_j} , \qquad \mathbf{K}_{12\cdots N} \equiv \mathbf{K}_{p_1} \mathbf{K}_{p_2} \cdots \mathbf{K}_{p_N} ,$$

and $\mathbf{S}_{12\cdots N}$ is the N-particle S matrix (which factorizes due to Yang-Baxter eq.).

Constraining the scalar factors

Scalar factor h in the hexagon $\longleftrightarrow$ dressing phase Σ in the S matrix

Constraining the scalar factors

Scalar factor h in the hexagon $\longleftrightarrow$ dressing phase Σ in the S matrix

We choose the **normalization** such that

- **Massive-massive**

$$h^{\bullet\bullet}(p,q) = \langle \mathbf{h}| Y_p Y_q \rangle = \langle \mathbf{h}| \tilde{Y}_p \tilde{Y}_q \rangle \,, \qquad \tilde{h}^{\bullet\bullet}(p,q) = \langle \mathbf{h}| Y_p \tilde{Z}_q \rangle = \langle \mathbf{h}| \tilde{Y}_p Z_q \rangle \,.$$

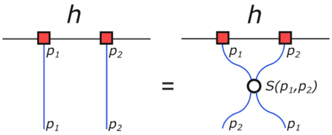
- **Massless-massless**

$$h^{\circ\circ}(p,q) = \langle \mathbf{h}| \chi_p^{\dot{A}} \chi_q^{\dot{B}} \rangle \,.$$

- **Mixed-mass**

$$h^{\bullet\circ}(p,q) = \langle \mathbf{h}| Y_p \chi_q^{\dot{A}} \rangle = \langle \mathbf{h}| \tilde{Y}_p \chi_q^{\dot{A}} \rangle \,, \qquad h^{\circ\bullet}(p,q) = \langle \mathbf{h}| \chi_p^{\dot{A}} Y_q \rangle = \langle \mathbf{h}| \chi_p^{\dot{A}} \tilde{Y}_q \rangle \,.$$

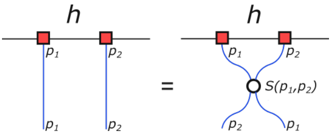
Constraining the scalar factors - Watson equation



Watson equation

Scattering two particles in the form factor with the full $\mathfrak{psu}(1|1)^{\oplus 4}$ S matrix $\mathbf{S}^{AdS_3 \times S^3 \times T^4}$ (with its dressing factors Σ), should not change the form factor.

Constraining the scalar factors - Watson equation



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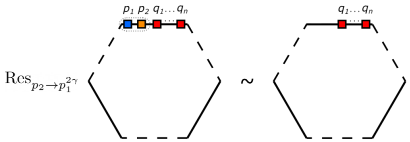
$$\frac{h^{\circ\circ}(p,q)}{h^{\circ\circ}(q,p)} = -\left[\Sigma^{\circ\circ}(p,q)\right]^2,$$

$$\frac{h^{\bullet\bullet}(p,q)}{h^{\bullet\bullet}(q,p)} = \left[\Sigma^{\bullet\bullet}(p,q)\right]^2, \qquad \frac{\tilde{h}^{\bullet\bullet}(p,q)}{\tilde{h}^{\bullet\bullet}(q,p)} = \left[\tilde{\Sigma}^{\bullet\bullet}(p,q)\right]^2,$$

$$\frac{h^{\bullet\circ}(p,q)}{h^{\circ\bullet}(q,p)} = \left[\Sigma^{\bullet\circ}(p,q)\right]^2, \qquad \frac{h^{\circ\bullet}(p,q)}{h^{\bullet\circ}(q,p)} = \left[\Sigma^{\circ\bullet}(p,q)\right]^2.$$

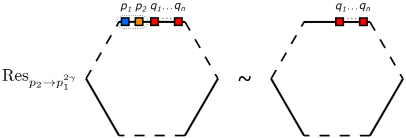
Constraining the scalar factors - Decoupling condition



Decoupling condition

Whenever two particles form a singlet, they decouple.

Constraining the scalar factors - Decoupling condition

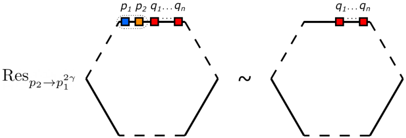


Decoupling condition

Whenever two particles form a singlet, they decouple.

$\rightarrow$ Singlet must be annihilated by momentum and energy operators
$\rightarrow$ $p_1 + p_2 = 0$ and $E(p_1) + E(p_2) = 0$
$\rightarrow$ *i.e.* one of the momenta is crossed $p_1 = p_2^{\pm 2\gamma}$.

Constraining the scalar factors - Decoupling condition



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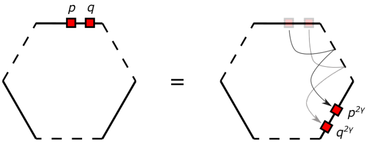
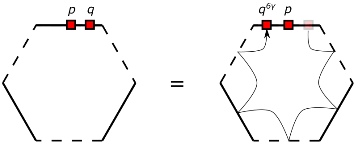
In the massless sector we find

$$h^{\circ\circ}(p,q)h^{\circ\circ}(p^{2\gamma},q) = e^{-\frac{i}{2}q}\,\frac{x^+_{\circ,p} - x^+_{\circ,q}}{x^+_{\circ,p} - x^-_{\circ,q}}\,.$$

Constraining the scalar factors - Cyclic invariance

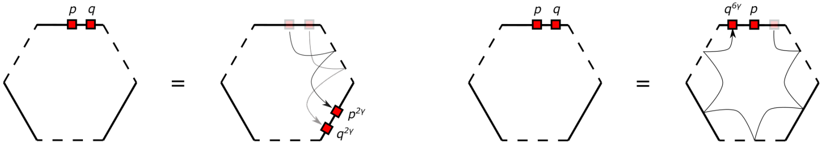




$$h^{oo}(p,q) = h^{oo}(p^{2\gamma}, q^{2\gamma})$$

$$\langle \mathbf{h} | \chi(p)\chi(q) \rangle = i \langle \mathbf{h} | \tilde{\chi}(q^{6\gamma})\chi(p) \rangle$$

Constraining the scalar factors - Cyclic invariance

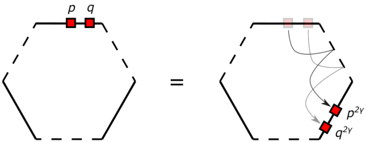


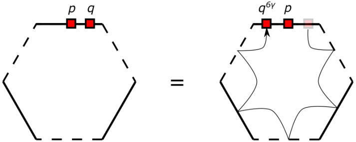
$$h^{\circ\circ}(p,q) = h^{\circ\circ}(p^{2\gamma}, q^{2\gamma}) \qquad \langle \mathbf{h} | \chi(p)\chi(q)\rangle = i \langle \mathbf{h} | \tilde{\chi}(q^{6\gamma})\chi(p)\rangle$$

This yields

$$h^{\circ\circ}(p,q) = e^{ip/2} \frac{x_\circ^-(p) - x_\circ^+(q)}{x_\circ^+(p) - x_\circ^+(q)}\, h^{\circ\circ}(q^{6\gamma}, p)\,.$$

Constraining the scalar factors - Cyclic invariance





$$h^{\circ\circ}(p,q) = h^{\circ\circ}(p^{2\gamma}, q^{2\gamma}) \qquad \langle \mathbf{h} | \chi(p)\chi(q) \rangle = i \langle \mathbf{h} | \tilde{\chi}(q^{6\gamma})\chi(p) \rangle$$

This yields

$$h^{\circ\circ}(p,q) = e^{ip/2} \frac{x_\circ^-(p) - x_\circ^+(q)}{x_\circ^+(p) - x_\circ^+(q)} \, h^{\circ\circ}(q^{6\gamma}, p) \,.$$

Using the crossing equation for $h^{\circ\circ}(p,q)$ leads to

$$h^{\circ\circ}(p,q) h^{\circ\circ}(q,p) = \frac{x_\circ^+(p) - x_\circ^+(q)}{x_\circ^+(p) - x_\circ^-(q)} \frac{x_\circ^-(p) - x_\circ^-(q)}{x_\circ^-(p) - x_\circ^+(q)} \,.$$

Solution for the scalar factors

- Watson equation

$$\frac{h^{\circ\circ}(p,q)}{h^{\circ\circ}(q,p)} = -\left[\Sigma^{\circ\circ}(p,q)\right]^2 = \frac{e^{\frac{i}{2}(p-q)}}{\sigma^{\circ\circ}(p,q)^2}\frac{x^-_{\circ,p} - x^+_{\circ,q}}{x^+_{\circ,p} - x^-_{\circ,q}}$$

- 6γ cyclic invariance

$$h^{\circ\circ}(p,q)h^{\circ\circ}(q,p) = \frac{x^+_\circ(p) - x^+_\circ(q)}{x^+_\circ(p) - x^-_\circ(q)}\frac{x^-_\circ(p) - x^-_\circ(q)}{x^-_\circ(p) - x^+_\circ(q)}$$

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- 6γ cyclic invariance

$$h^{\circ\circ}(p,q)h^{\circ\circ}(q,p) = \frac{x^+_\circ(p) - x^+_\circ(q)}{x^+_\circ(p) - x^-_\circ(q)}\frac{x^-_\circ(p) - x^-_\circ(q)}{x^-_\circ(p) - x^+_\circ(q)}$$

Solutions

$$h^{\circ\circ}(p,q) = \frac{e^{i(p-q)/4}}{\sigma^{\circ\circ}(p,q)}\sqrt{\frac{[x^+_\circ(p) - x^+_\circ(q)][x^-_\circ(p) - x^-_\circ(q)]}{[x^+_\circ(p) - x^-_\circ(q)]^2}}\,.$$

In a similar way, solutions can be found for $h^{\bullet\bullet}(p,q)$, $\tilde{h}^{\bullet\bullet}(p,q)$, $h^{\bullet\circ}(p,q)$, $h^{\circ\bullet}(p,q)$.

Plan

1. Review of integrability for $AdS_3 \times S^3 \times T^4$
2. Integrability for three-point functions and the hexagon operator
3. **Explicit Check**
4. Conclusions and outlook

Protected multiplets

- Left and right R-charge $(J, \tilde{J})$
- For every positive integer j:

$$
\begin{array}{ccc}
 & \left(\frac{j-1}{2}, \frac{j-1}{2}\right) & \\
 \left(\frac{j}{2}, \frac{j-1}{2}\right)^{\dot{A}} & & \left(\frac{j-1}{2}, \frac{j}{2}\right)^{\dot{A}} \\
\left(\frac{j+1}{2}, \frac{j-1}{2}\right) & \left(\frac{j}{2}, \frac{j}{2}\right)^{\dot{A}\dot{B}} & \left(\frac{j-1}{2}, \frac{j+1}{2}\right) \\
 \left(\frac{j+1}{2}, \frac{j}{2}\right)^{\dot{A}} & & \left(\frac{j}{2}, \frac{j+1}{2}\right)^{\dot{A}} \\
 & \left(\frac{j+1}{2}, \frac{j+1}{2}\right) &
\end{array}
$$

- BPS multiplets can mix among each other as j changes
- $\left(\frac{j+1}{2}, \frac{j-1}{2}\right) \leftrightarrow \mathbb{V}_j^{+-}$ and $\left(\frac{j-1}{2}, \frac{j+1}{2}\right) \leftrightarrow \mathbb{V}_j^{-+}$ do not mix

Protected multiplets

We have $\left(\frac{j+1}{2}, \frac{j-1}{2}\right) \leftrightarrow \mathbb{V}_j^{+-}$ and $\left(\frac{j-1}{2}, \frac{j+1}{2}\right) \leftrightarrow \mathbb{V}_j^{-+}$

Magnon	$\mathbf{J}^3$	$\widetilde{\mathbf{J}}^3$
$\lim_{p\to 0^+} \lvert\chi^1(p)\rangle$	$-\frac{1}{2}$	0
$\lim_{p\to 0^-} \lvert\chi^2(p)\rangle$	0	$+\frac{1}{2}$
$\lim_{p\to 0^+} \lvert\tilde{\chi}^1(p)\rangle$	$+\frac{1}{2}$	0
$\lim_{p\to 0^-} \lvert\tilde{\chi}^2(p)\rangle$	0	$-\frac{1}{2}$

[Dei, Sfondrini '18]

Protected multiplets

We have $\left(\frac{j+1}{2}, \frac{j-1}{2}\right) \leftrightarrow \mathbb{V}_j^{+-}$ and $\left(\frac{j-1}{2}, \frac{j+1}{2}\right) \leftrightarrow \mathbb{V}_j^{-+}$

Magnon	$\mathbf{J}^3$	$\widetilde{\mathbf{J}}^3$
$\lim_{p\to 0^+} \lvert\chi^1(p)\rangle$	$-\frac{1}{2}$	0
$\lim_{p\to 0^-} \lvert\chi^2(p)\rangle$	0	$+\frac{1}{2}$
$\lim_{p\to 0^+} \lvert\tilde{\chi}^1(p)\rangle$	$+\frac{1}{2}$	0
$\lim_{p\to 0^-} \lvert\tilde{\chi}^2(p)\rangle$	0	$-\frac{1}{2}$

[Dei, Sfondrini '18]

Identify

$$\mathbb{V}_j^{+-} \sim \lim_{p\to 0^+} \lvert\tilde{\chi}^1(p)\tilde{\chi}^2(-p)\rangle \,, \qquad \mathbb{V}_j^{-+} \sim \lim_{p\to 0^+} \lvert\chi^1(p)\chi^2(-p)\rangle \,.$$

The correlation functions

$$\langle \mathbb{V}^{-+}_{j_1} \mathbb{V}^{-+}_{j_2} \mathbb{V}^{-+}_{j_3} \rangle = -\frac{1}{4\sqrt{N}} D_{J_1 J_2 J_3} D_{\tilde{J}_1 \tilde{J}_2 \tilde{J}_3} \frac{(j_1 + j_2 + j_3 - 1)(j_1 + j_2 + j_3 + 1)}{\sqrt{j_1 j_2 j_3}},$$

$$\langle \mathbb{V}^{-+}_{j_1} \mathbb{V}^{-+}_{j_2} \mathbb{V}^{+-}_{j_3} \rangle = -\frac{1}{4\sqrt{N}} D_{J_1 J_2 J_3} D_{\tilde{J}_1 \tilde{J}_2 \tilde{J}_3} \frac{(j_1 + j_2 - j_3 - 1)(j_1 + j_2 - j_3 + 1)}{\sqrt{j_1 j_2 j_3}}.$$

[Gaberdiel, Kirsch '07] [Dabholkar, Pakman '07] [Pakman, Sever '07]

The correlation functions

$$\langle \mathbb{V}^{-+}_{j_1}\mathbb{V}^{-+}_{j_2}\mathbb{V}^{-+}_{j_3}\rangle = -\frac{1}{4\sqrt{N}} D_{J_1J_2J_3} D_{\tilde{J}_1\tilde{J}_2\tilde{J}_3} \frac{(j_1+j_2+j_3-1)(j_1+j_2+j_3+1)}{\sqrt{j_1j_2j_3}},$$

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[Gaberdiel, Kirsch '07] [Dabholkar, Pakman '07] [Pakman, Sever '07]

Using the hexagon formalism

$$\langle \mathbb{V}^{-+}_{j_1}\mathbb{V}^{-+}_{j_2}\mathbb{V}^{-+}_{j_3}\rangle \sim \Big(\{\chi^1(p_1),\chi^2(-p_1)\},\ \{\chi^1(p_2),\chi^2(-p_2)\},\ \{\chi^1(p_3),\chi^2(-p_3)\}\Big)$$

$$\sim \Big(\prod_{j=1}^{3}\sum_{X_j=\alpha_j\cup\bar{\alpha}_j}(-1)^{\bar{\alpha}_j} w_{\alpha_j,\bar{\alpha}_j}\Big)\langle \mathbf{h}|\alpha_1^{4\gamma}\alpha_2^{2\gamma}\alpha_3^{0\gamma}\rangle\,\langle \mathbf{h}|\bar{\alpha}_2^{4\gamma}\bar{\alpha}_1^{2\gamma}\bar{\alpha}_3^{0\gamma}\rangle\,.$$

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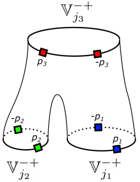
$$\sim \Big(\prod_{j=1}^{3} \sum_{X_j = \alpha_j \cup \bar{\alpha}_j} (-1)^{\bar{\alpha}_j} w_{\alpha_j, \bar{\alpha}_j} \Big) \langle \mathbf{h} | \alpha_1^{4\gamma} \alpha_2^{2\gamma} \alpha_3^{0\gamma} \rangle \, \langle \mathbf{h} | \bar{\alpha}_2^{4\gamma} \bar{\alpha}_1^{2\gamma} \bar{\alpha}_3^{0\gamma} \rangle .$$

Similarly

$$\langle \mathbb{V}^{-+}_{j_1} \mathbb{V}^{-+}_{j_2} \mathbb{V}^{+-}_{j_3} \rangle \sim \Big(\{\chi^1(p_1), \chi^2(-p_1)\}, \{\chi^1(p_2), \chi^2(-p_2)\}, \{\tilde{\chi}^1(p_3), \tilde{\chi}^2(-p_3)\} \Big) .$$

Cutting three-point Functions

$$\langle \mathbb{V}_{j_1}^{-+}\mathbb{V}_{j_2}^{-+}\mathbb{V}_{j_3}^{-+}\rangle \sim \left(\prod_{j=1}^{3} \sum_{X_j=\alpha_j\cup\bar{\alpha}_j} (-1)^{\bar{\alpha}_j} w_{\alpha_j,\bar{\alpha}_j}\right)\langle \mathbf{h}|\alpha_1^{4\gamma}\alpha_2^{2\gamma}\alpha_3^{0\gamma}\rangle \, \langle \mathbf{h}|\bar{\alpha}_2^{4\gamma}\bar{\alpha}_1^{2\gamma}\bar{\alpha}_3^{0\gamma}\rangle$$



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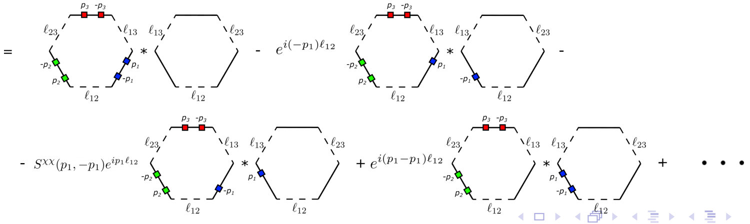

Limit procedure

- Particles in each state need to satisfy the Bethe equations

$$e^{ip_k j_k} = 1 \qquad \Rightarrow \qquad p_k = \frac{2\pi\nu_k}{j_k}\,, \quad \nu_k \in \mathbb{Z}\,, \qquad k = 1, 2, 3\,.$$

- Introduce $\varepsilon > 0$ and real numbers ϵ_k and redefine

$$j_k \to \frac{j_k}{1+\varepsilon\,\epsilon_k}\,, \qquad p_k \to p_k(1+\varepsilon\,\epsilon_k)\,.$$

Coincident-momenta limit by setting $p_1 = p_2 = p_3$ with $\varepsilon \to 0$.

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Coincident-momenta limit by setting $p_1 = p_2 = p_3$ with $\varepsilon \to 0$.

- Correlation functions are protected for *any* value of h, k.

[Baggio, de Boer, Papadodimas '12]

- Kinematics of massless particles only depends on h/k.

Result

- Evaluation of $\langle \mathbb{V}_{j_1}^{-+} \mathbb{V}_{j_2}^{-+} \mathbb{V}_{j_3}^{-+} \rangle$ for small p leads to

$$4\left(j_1 + j_2 + j_3 - 1\right)\left(j_1 + j_2 + j_3 + 1\right) p^2 + \dots$$

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- Evaluation of $\langle \mathbb{V}_{j_1}^{-+} \mathbb{V}_{j_2}^{-+} \mathbb{V}_{j_3}^{+-} \rangle$ for small p leads to

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$$4\left(j_1+j_2-j_3-1\right)\left(j_1+j_2-j_3+1\right)p^2+\ldots$$

Ratio matches!

$$\frac{\langle \mathbb{V}_{j_1}^{-+}\mathbb{V}_{j_2}^{-+}\mathbb{V}_{j_3}^{-+}\rangle}{\langle \mathbb{V}_{j_1}^{-+}\mathbb{V}_{j_2}^{-+}\mathbb{V}_{j_3}^{+-}\rangle}=\frac{\left(j_1+j_2+j_3-1\right)\left(j_1+j_2+j_3+1\right)}{\left(j_1+j_2-j_3-1\right)\left(j_1+j_2-j_3+1\right)}$$

Plan

Conclusions

- **Hexagon approach** for the computation of three-point functions by integrability can also be applied to $\mathbf{AdS_3 \times S^3 \times T^4}$
- Structure entirely **fixed by symmetry** up to dressing factors
- In principal valid for pure RR and mixed flux
- First string theory other than the original $AdS_5 \times S^5$
- **Successful check** of the resulting machinery with protected states

Outlook

- Study **mixture of NSNS** and **RR** background fluxes
  → Dressing factors? Highly non-trivial! [Babichenko, Dekel, Ohlsson Sax '14]

- Study **pure NSNS** limit
  → limit is quite tricky
  → wrapping should be simple and possible to compute results by CFT [Dei, Eberhardt '20, '21]

- Study **pure RR** limit
  → dressing factors already proposed
  → cannot compare with CFT results

- **Higher-point** correlation functions and **non-planar** correlators
  → in particular four-points four BPS operators

- Which **other backgrounds** are amenable to this bootstrap approach?