S matrices for quantum-deformed superstrings

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Integrability in Gauge and String Theory 2021

Imperial College London

FNSNF SWISS NATIONAL SCIENCE FOUNDATION

Mainly based on 1811.07841 with B. Hoare
2007.09136 & 2107.02564 with S. van Tongeren and Y. Zimmermann

Motivation

- Integrability in the AdS/CFT correspondence

type IIB superstrings on $\mathrm{AdS}_5 \times \mathrm{S}^5$ (free strings)	$\Leftrightarrow$	$\mathcal{N} = 4$ Super-Yang-Mills in 4D Minkowski space (planar graphs)

  - string spectrum / anomalous dimension of operators
  - Structure constants, higher-point correlation functions

- Are there other superstring theories for which the integrability program can be applied?

Motivation

- Lower dimensional AdS/CFT setups

$$\mathrm{AdS}_4 \times \mathbb{CP}^3$$

$$\mathrm{AdS}_3 \times S^3 \times T^4 \qquad \mathrm{AdS}_3 \times S^3 \times S^3 \times S^1$$

$$\mathrm{AdS}_2 \times S^2 \times T^6 \qquad \mathrm{AdS}_2 \times S^2 \times S^2 \times T^4$$

  - Less supersymmetries, new features (massless modes, B-field, moduli)
  - Still the most supersymmetric in their respective dimension

Motivation

- **Integrable deformations** of AdS/CFT setups

TsT transformations

Motivation

- **Integrable deformations** of AdS/CFT setups

TsT transformations

- T-duality, shift, T-duality
- maps string theory to string theory
- In some cases dual is known [Leigh Strassler '95] [Lunin Maldacena '05]

Motivation

- **Integrable deformations** of AdS/CFT setups

TsT transformations

$T\bar{T}$ deformations, more general current-current deformations

Quantum deformations (η-deformations, λ-deformations)

Yang-Baxter deformations

Motivation

- **Integrable deformations** of AdS/CFT setups

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Yang-Baxter deformations

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Semi-symmetric space sigma model

• On the string side of the duality, classical integrability arises from the formulation of the Green-Schwarz string as a sigma model on a semi-symmetric space.

[Metsaev Tseytlin '98] [Bena Polchinski Roiban '04]

Semi-symmetric space sigma model

• On the string side of the duality, classical integrability arises from the formulation of the Green-Schwarz string as a sigma model on a semi-symmetric space.

[Metsaev Tseytlin '98] [Bena Polchinski Roiban '04]

$=$ sigma model on a supercoset G/H, where $\mathfrak{g} = \mathrm{Lie(G)}$ has a $\mathbb{Z}_4$ grading

even grading   odd grading

$$\mathfrak{g} = \mathfrak{g}^{(0)} + \mathfrak{g}^{(1)} + \mathfrak{g}^{(2)} + \mathfrak{g}^{(3)}$$

subalgebra $\mathfrak{h} = \mathrm{Lie(H)}$

Semi-symmetric space sigma model

Weyl-invariant metric on worldsheet

String tension

ad-invariant bilinear form on $\mathfrak{g} = \mathrm{Lie}(\mathrm{G})$

supergroup-valued field $g \in \mathrm{G}$

$$S = T \int \mathrm{d}\tau \mathrm{d}\sigma (\gamma^{ij} - \epsilon^{ij}) \, \mathrm{STr} \left[g^{-1} \partial_i g \, P \, g^{-1} \partial_j g \right]$$

antisymmetric tensor

$P = P^{(2)} + \frac{1}{2} \left(P^{(1)} - P^{(3)} \right)$

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- Global left acting G symmetry $g \to g_0 g$
- Local right acting H symmetry $g \to gh$
- Worldsheet diffeomorphisms, fermionic κ-symmetry
- Classical integrability: equations of motion $\Leftrightarrow$ flat Lax connection

[Bena Polchinski Roiban '04]

Semi-symmetric space sigma model G/H

- Describes Green-Schwarz superstrings on various backgrounds

$$\frac{\mathsf{PSU}(2,2|4)}{\mathsf{SO}(1,4)\times\mathsf{SO}(5)} \quad\Updownarrow\quad \mathsf{AdS}_5\times \mathsf{S}^5$$

$$\frac{\mathsf{Osp}(6|4)}{\mathsf{U}(3)\times\mathsf{SL}(2;\mathbb{C})} \quad\Updownarrow\quad \mathsf{AdS}_4\times \mathbb{C}\mathsf{P}^3$$

$$\frac{\mathsf{PSU}(1,1|2)}{\mathsf{SO}(1,1)\times\mathsf{SO}(2)} \quad\Updownarrow\quad \mathsf{AdS}_2\times \mathsf{S}^2\times \mathsf{T}^6$$

$$\frac{\mathsf{PSU}(1,1|2)\times\mathsf{PSU}(1,1|2)}{\mathsf{SU}(1,1)\times\mathsf{SU}(2)} \quad\Updownarrow\quad \mathsf{AdS}_3\times \mathsf{S}^3\times \mathsf{T}^4$$

$$\frac{\mathsf{D}(2,1;\alpha)\times\mathsf{D}(2,1;\alpha)}{\mathsf{SU}(1,1)\times\mathsf{SU}(2)\times\mathsf{SU}(2)} \quad\Updownarrow\quad \mathsf{AdS}_3\times \mathsf{S}^3\times \mathsf{S}^3\times \mathsf{T}^1$$

Ingredients of the deformation

- Deformation parameter $\eta \in [0,1)$
- **Drinfel'd Jimbo** operator $R : \mathfrak{g} \to \mathfrak{g}$

$$\mathfrak{g} = \{H, E^{+}, E^{-}\} \qquad R(H) = 0 \qquad R(E^{\pm}) = \pm i\, E^{\pm}$$

  - Antisymmetric with respect to the supertrace

$$\mathrm{STr}[R(X)\, Y] = -\mathrm{STr}[X\, R(Y)]$$

  - Satisfies the inhomogeneous classical Yang-Baxter equation

$$[R(X), R(Y)] - R\left([R(X), Y] + [X, R(Y)]\right) = [X, Y]$$

Deformed semi-symmetric space sigma model

[Klimcik '02 '08]
[Delduc Magro Vicedo '13 '14]

$$P_\eta = P^{(2)} + \tfrac{1-\eta^2}{2}\left(P^{(1)} - P^{(3)}\right)$$

$$S = T \int \mathrm{d}\tau \mathrm{d}\sigma (\gamma^{ij} - \epsilon^{ij}) \,\mathrm{STr}\left[g^{-1}\partial_i g \, P_\eta \frac{1}{1 - \frac{2\eta}{1-\eta^2} R_g P_\eta} g^{-1}\partial_j g \right]$$

$$R_g(X) = g^{-1} R(g X g^{-1}) g$$

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- Global left acting G symmetry $g \to g_0 g$ ✗ broken to U(1)s
- Local right acting H symmetry $g \to gh$ ✓
- Worldsheet diffeomorphisms, fermionic κ-symmetry ✓
- Classical integrability: equations of motion $\Leftrightarrow$ flat Lax connection ✓

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- Local right acting H symmetry $g \to gh$ ✓
- Worldsheet diffeomorphisms, fermionic κ-symmetry ✓
- Classical integrability: equations of motion $\Leftrightarrow$ flat Lax connection ✓
- Quantum group symmetry $\mathfrak{g}_q$ $\qquad q = \exp\left[-\frac{2\eta}{1-\eta^2}\frac{1}{T}\right]$

Is the model unique?

- **Drinfel'd Jimbo** operator $R : \mathfrak{g} \to \mathfrak{g}$

$$\mathfrak{g} = \{H, E^+, E^-\} \qquad R(H) = 0 \qquad R(E^\pm) = \pm i\, E^\pm$$

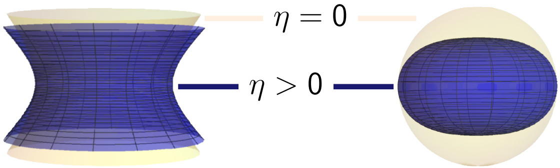
  - $\mathfrak{g}$ is a complex semi-simple Lie algebra or compact real form. Ex: $\mathfrak{sl}(4)$, $\mathfrak{su}(4)$.
    unique $R \to$ unique η-deformation
  - $\mathfrak{g}$ is a non-compact real semi-simple Lie algebra. Ex: $\mathfrak{su}(2,2)$.
    $\neq R \to \eta$-deformations related by complex field redefinition
  - $\mathfrak{g}$ is a complex semi-simple Lie superalgebra. Ex: $\mathfrak{sl}(2|2)$.
    $\neq R$, associated to $\neq$ Dynkin diagrams $\to \neq \eta$-deformations

Deformed backgrounds

- Choose parametrisation, expand to quadratic order in fermions, compare to GS string

$$\mathcal{G}^{(R)}_{\mu\nu}(\eta) \qquad \mathcal{B}^{(R)}_{\mu\nu}(\eta) \qquad \mathcal{F}^{(R)}(\eta)$$

  - Same metric, B-field (up to complex field redefinition [Hoare van Tongeren '16])



  - Different Ramond-Ramond fluxes [Borsato Wulff '16] [Hoare **FS** '18]

Ex: Deformations of $AdS_5 \times S^5$

- Symmetry algebra $\mathfrak{psu}(2,2|4)$.

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  - R_d 

[Arutyunov Borsato Frolov '13 '15]
[Arutyunov Frolov Hoare Roiban Tseytlin '15]
[Tseytlin Wulff '15]

$$\mathcal{F}_1(\eta) + \mathcal{F}_3(\eta) + \mathcal{F}_5(\eta) \longrightarrow \text{Not SUGRA}$$

Ex: Deformations of $AdS_5 \times S^5$

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  - R_f 

    [Hoare **FS** '18]

    $\mathcal{F}_3(\eta) + \mathcal{F}_5(\eta) \longrightarrow$ SUGRA

Ex: Deformations of (pure RR) $AdS_3 \times S^3 \times T^4$

- Symmetry algebra $\mathfrak{psu}(1,1|2)_\mathsf{L} \oplus \mathfrak{psu}(1,1|2)_\mathsf{R}$. Can define a 2-parameter deformation.

[Klimcik '14] [Hoare '14][**FS** '19]

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[Klimcik '14] [Hoare '14][**FS** '19]

  - R_{oxo} $\quad [\bigcirc\!-\!\otimes\!-\!\bigcirc]^{\oplus 2}$

    $\mathcal{F}_1(\eta_{\mathrm{L,R}}) + \mathcal{F}_3(\eta_{\mathrm{L,R}}) + \mathcal{F}_5(\eta_{\mathrm{L,R}}) \longrightarrow$ Not SUGRA

  - R_{xox} $\quad [\otimes\!-\!\bigcirc\!-\!\otimes]^{\oplus 2}$

    $\mathcal{F}_1(\eta_{\mathrm{L,R}}) + F_3(\eta_{\mathrm{L,R}}) + \mathcal{F}_5(\eta_{\mathrm{L,R}}) \longrightarrow$ Not SUGRA

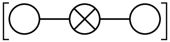
  - R_{xxx} $\quad [\otimes\!-\!\otimes\!-\!\otimes]^{\oplus 2}$

    $\mathcal{F}_3(\eta_{\mathrm{L,R}}) + \mathcal{F}_5(\eta_{\mathrm{L,R}}) \longrightarrow$ SUGRA

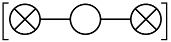
Ex: Deformations of (pure RR) $AdS_3 \times S^3 \times T^4$

- Symmetry algebra $\mathfrak{psu}(1,1|2)_L \oplus \mathfrak{psu}(1,1|2)_R$. Can define a 2-parameter deformation.

[Klimcik '14] [Hoare '14][**FS** '19]

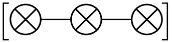
  - R_{oxo}  $^{\oplus 2}$

    $\mathcal{F}_1(\eta_{L,R}) + \mathcal{F}_3(\eta_{L,R}) + \mathcal{F}_5(\eta_{L,R})$ ⟶ Not SUGRA

  - R_{xox}  $^{\oplus 2}$

    $\mathcal{F}_1(\eta_{L,R}) + F_3(\eta_{L,R}) + \mathcal{F}_5(\eta_{L,R})$ ⟶ Not SUGRA

$\eta_{L,R}$-dependent TsT in torus direction

  - R_{xxx}  $^{\oplus 2}$

    $\mathcal{F}_3(\eta_{L,R}) + \mathcal{F}_5(\eta_{L,R})$ ⟶ SUGRA

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Symmetries: Drinfel'd type quantum group

- Lie superalgebra $\mathfrak{g} = \{H, E^+, E^-\}$

$$[H_j, E_k^{\pm}] = \pm A_{jk} E_k^{\pm}$$

$$[E_j^+, E_k^-] = \delta_{jk} H_k$$

$$\Delta(X) = X \otimes 1 + 1 \otimes X$$

$$X \in \{H_j, E_j^{\pm}\}$$

Coproduct

$$\Delta([X, Y]) = [\Delta(X), \Delta(Y)]$$

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- Drinfel'd quantum group $\mathfrak{g}_q$

$$[H_j, E_k^{\pm}] = \pm A_{jk} E_k^{\pm}$$
$$[E_j^+, E_k^-] = \delta_{jk} \frac{q^{H_j} - q^{-H_j}}{q - q^{-1}}$$

$$\Delta(H_j) = H_j \otimes 1 + 1 \otimes H_j$$
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Symmetries: Drinfel'd type quantum group

- Lie superalgebra $\mathfrak{g} = \{H, E^+, E^-\}$ ⟵ The Cartan-Weyl basis chosen to construct the Drinfel'd Jimbo operator R

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Symmetries: Drinfel'd type quantum group

$\mathfrak{g}$

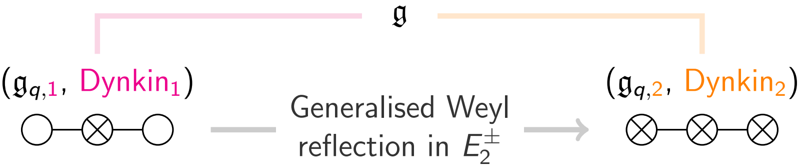
$(\mathfrak{g}_{q,1}, \text{Dynkin}_1)$ $(\mathfrak{g}_{q,2}, \text{Dynkin}_2)$

- q-deformed algebras are isomorphic $\omega([X, Y]) = [\omega(X), \omega(Y)]$
- q-deformed coproducts are related by a **twist** $(\omega \otimes \omega)\Delta(X) = F^{-1}\Delta'(\omega(X))F$

[Seganova '85] [Khoroshkin Tolstoy '91 '94]

$$(F \otimes 1)(\Delta \otimes 1)F = (1 \otimes F)(1 \otimes \Delta)F$$

Symmetries: Drinfel'd type quantum group



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$$(F \otimes 1)(\Delta \otimes 1)F = (1 \otimes F)(1 \otimes \Delta)F$$

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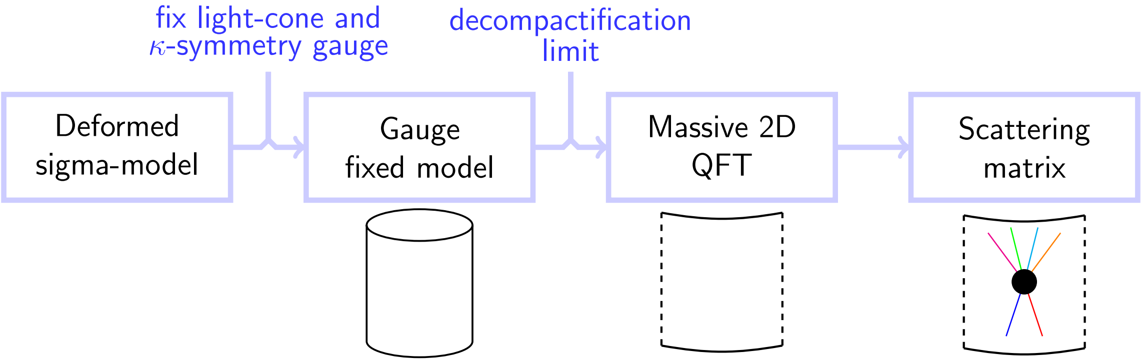
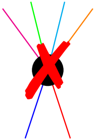
Finding the S matrix



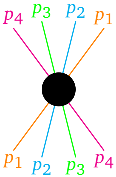
- Assumptions
  - The q-deformed symmetry extends to the quantum theory
  - Classical integrability extends to quantum integrability

- No particle production

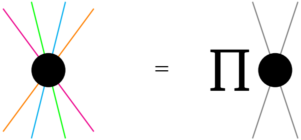


[Zamolodchikov Zamolodchikov '79]

- Transmitted momenta

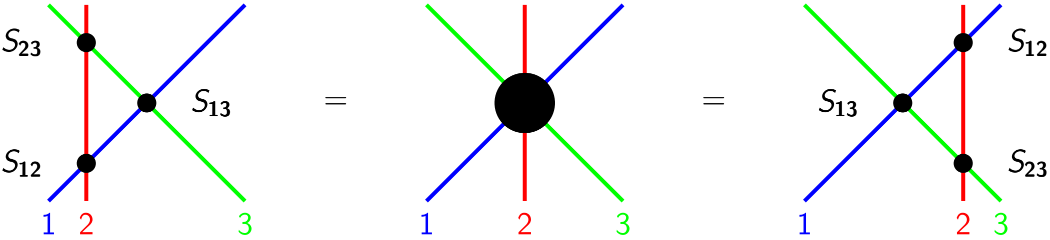


- Factorisation



Integrable S matrices

- For integrable 2D QFT the basic building bloc is the two-body S matrix
  - Satisfies the **quantum Yang-Baxter equation**



  - Bootstrapped using the **q-deformed symmetry** of the quantum theory

$AdS_5 \times S^5$: Light-cone gauge symmetries

- Symmetry breaking pattern

$$\mathfrak{psu}(2,2|4) \qquad \rightarrow \qquad \mathcal{A} = [\mathfrak{su}(2|2) \oplus \mathfrak{su}(2|2)]_{\text{c.e.}}$$

$\mathrm{AdS}_5 \times S^5$: Light-cone gauge symmetries

- Symmetry breaking pattern

$$\mathfrak{psu}_q(2,2|4) \qquad \rightarrow \qquad \mathcal{A} = \left[\mathfrak{su}_q(2|2) \oplus \mathfrak{su}_{1/q}(2|2)\right]_{\text{c.e.}}$$

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  - R_d  $\rightarrow$ 

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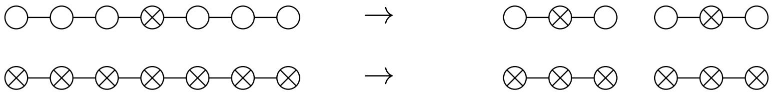
- R_{d}
- R_{f}



AdS$_5\times$ S^5: Light-cone gauge symmetries

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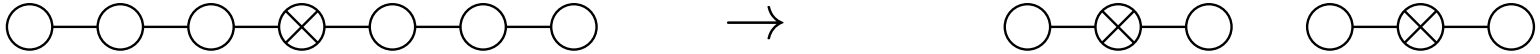
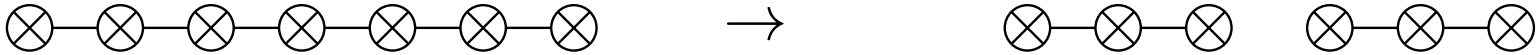
  - R_d 
  - R_f 

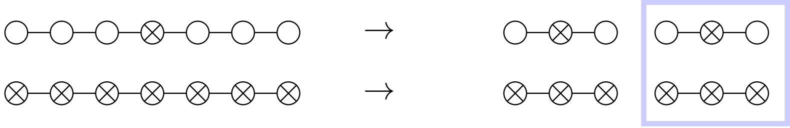
- Sufficient to construct the $\mathfrak{su}_q(2|2)_{\text{c.e.}}$ S matrices

$AdS_5 \times S^5$: Light-cone gauge symmetries

- Symmetry breaking pattern

$$\mathfrak{psu}_q(2,2|4) \quad \to \quad \mathcal{A} = \left[\mathfrak{su}_q(2|2) \oplus \mathfrak{su}_{1/q}(2|2)\right]_{\text{c.e.}}$$

  - R_d
  - R_f



- Sufficient to construct the $\mathfrak{su}_q(2|2)_{\text{c.e.}}$ S matrices
- They are related by a twist

$$S_f = F_{op}^{-1} S_d F \qquad F = \exp\left[(q - q^{-1}) U^{1/2} E_2^{+} \otimes U^{1/2} E_2^{-}\right]$$

$AdS_5 \times S^5$: Fundamental $\mathfrak{su}_q(2|2)_{\text{c.e.}}$ S matrices

- The distinguished S matrix [Beisert Koroteev '08] [Beisert Galleas Matsumoto '11]
  - Braiding and matrix unitarity, solves the qYBE, can impose crossing symmetry
  - There is a bosonic $\mathfrak{su}_q(2) \oplus \mathfrak{su}_q(2)$ sub-Hopf algebra

$$S = \begin{pmatrix} S_{\mathfrak{su}_q(2)} & * & * \\ * & S_{\mathfrak{su}_q(2)} & * \\ * & * & * \end{pmatrix} \qquad S_{\mathfrak{su}_q(2)} = \begin{pmatrix} A & 0 & 0 & 0 \\ 0 & \frac{A-B}{q+q^{-1}} & \frac{qA+q^{-1}B}{q+q^{-1}} & 0 \\ 0 & \frac{q^{-1}A+qB}{q+q^{-1}} & \frac{A-B}{q+q^{-1}} & 0 \\ 0 & 0 & 0 & A \end{pmatrix}$$

  - Tree-level expansion matches perturbative calculations if using R_{d} and

$$q = \exp\left[-\frac{2\eta}{1-\eta^2}\frac{1}{T}\right]$$

[Arutyunov Borsato Frolov '13]
[**FS** van Tongeren Zimmermann '20]

$\mathrm{AdS}_5\times \mathrm{S}^5$: Fundamental $\mathfrak{su}_q(2|2)_{\text{c.e.}}$ S matrices

- The fermionic S matrix [**FS** van Tongeren Zimmermann '20]
  - Braiding and matrix unitarity, solves the qYBE, same crossing equations
  - $\mathfrak{su}_q(2)\oplus\mathfrak{su}_q(2)$ is no longer a sub-Hopf algebra

$$S_{\mathfrak{su}_q(2)} = \begin{pmatrix} A & 0 & 0 & 0 \\ 0 & \frac{A-B}{q+q^{-1}} & \frac{qA+q^{-1}B}{q+q^{-1}} & 0 \\ 0 & \frac{q^{-1}A+qB}{q+q^{-1}} & \frac{A-B}{q+q^{-1}} & 0 \\ 0 & 0 & 0 & A \end{pmatrix} \to \begin{pmatrix} A & 0 & 0 & 0 \\ 0 & \frac{A-B}{q+q^{-1}} & * & 0 \\ 0 & * & \frac{A-B}{q+q^{-1}} & 0 \\ 0 & 0 & 0 & A \end{pmatrix}$$

  - Tree-level expansion matches perturbative calculations if using R_f and same q
  - Not related to distinguished S matrix by a one particle change of basis

$AdS_5 \times S^5$: The twist

- The two S matrices are related by a twist [FS van Tongeren Zimmermann '20]

$$S_f = F_{op}^{-1} S_d F \qquad F = \exp\left[(q - q^{-1}) U^{1/2} E_2^+ \otimes U^{1/2} E_2^-\right]$$

$$\begin{aligned}
(F-1)\,|\phi_1\phi_2\rangle &= +(q-q^{-1})U_1^{1/2}U_2^{1/2}c_1 a_2\,|\psi_3\psi_4\rangle\\
(F-1)\,|\phi_1\psi_3\rangle &= +(q-q^{-1})U_1^{1/2}U_2^{1/2}c_1 b_2\,|\psi_3\phi_1\rangle\\
(F-1)\,|\psi_4\phi_2\rangle &= -(q-q^{-1})U_1^{1/2}U_2^{1/2}d_1 a_2\,|\phi_2\psi_4\rangle\\
(F-1)\,|\psi_4\psi_3\rangle &= -(q-q^{-1})U_1^{1/2}U_2^{1/2}d_1 b_2\,|\phi_2\phi_1\rangle
\end{aligned}$$

braiding — representation parameters

$AdS_3 \times S^3 \times T^4$ (pure RR, massive modes)

- Symmetry breaking pattern

$$\mathfrak{psu}(1,1|2)_{\mathsf{L}} \oplus \mathfrak{psu}(1,1|2)_{\mathsf{R}} \;\rightarrow \mathcal{A} = [\mathfrak{su}(1|1)_{\mathsf{L}} \oplus \mathfrak{su}(1|1)_{\mathsf{R}}]^{\oplus 2}_{\mathsf{c.e.}}$$

$AdS_3 \times S^3 \times T^4$ (pure RR, massive modes)

- Symmetry breaking pattern

$$\mathfrak{psu}_{q_L}(1,1|2) \oplus \mathfrak{psu}_{q_R}(1,1|2) \;\rightarrow \mathcal{A} = [\mathfrak{su}_{q_L}(1|1) \oplus \mathfrak{su}_{q_R}(1|1)]^{\oplus 2}_{\text{c.e.}}$$

$\mathrm{AdS}_3 \times \mathrm{S}^3 \times \mathrm{T}^4$ (pure RR, massive modes)

- Symmetry breaking pattern

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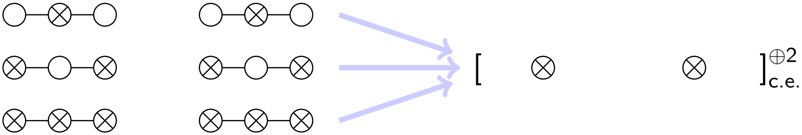

$AdS_3 \times S^3 \times T^4$ (pure RR, massive modes)

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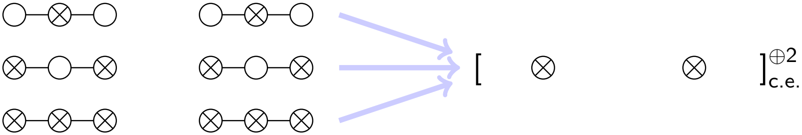

  - The fundamental (q_L, q_R)-deformed S matrix has been constructed [Hoare '14]
  - Tree-level expansion matches with calculations from perturbation theory

[Bocconcello Masuda **FS** Sfondrini '20] [**FS** van Tongeren Zimmermann '21]

Table of Contents

Conclusions

- How to generate new integrable theories? **Integrable deformations!**
- Ex: η-deformations, conjectured to correspond to a **q-deformation**, promoting the symmetry algebra to a Drinfel'd type quantum group
- η-deformations of superstrings / q-deformations of superalgebras **are not unique**
- **Only one** η-deformation is a supergravity solution
- The exact q-deformed S matrices are related by a **twist**
- Matching between exact q-deformed S-matrix and perturbative S-matrix
- Compatibillity with integrability

Outlook

- How does the twist affect **physical observables**: Bethe equations, QSC

  [Beisert Koroteev '08] [Arutyunov de Leeuw van Tongeren '14] [Klabbers van Tongeren '17]

- Interpretation of the twist at the level of background

- Better understand the **Weyl anomaly** in η-deformations

  [Fernandez-Melgarejo Sakamoto Sakatani Yoshida '17] [Sakamoto Sakatani Yoshida '18] [Mück '19]

- Connection between η- and λ-deformations: **Poisson-Lie duality**

  [Klimcik Severa '95]
  [Sfetsos '13] [Hollowood Miramontes Schmidtt '14]
  [Hoare Tseytlin '15] [Sfetsos Siampos Thompson '15]

- q-deformations in the context of **holography**

Thank You!